Review of *Recent Results in the Theory of Graph Spectra* by D. Cvetković, M. Doob, I. Gutman, and A. Torĝasev*

Richard A. Brualdi
Department of Mathematics
University of Wisconsin
Madison, Wisconsin 53706

Spectral graph theory is concerned with the relationship between the algebraic properties of the spectra of a matrix associated with a graph (usually, but not always, this matrix is the adjacency matrix) and the topological properties of the graph. The book [1] contains almost all results on this subject which were known as of 1978. The present book reviews results which have appeared since 1978. Proofs are either omitted or outlined briefly. But there is a bibliography of over 700 items which alone is worth the price of the book.

One of the most striking theorems in spectral graph theory is the following theorem of Cameron, Goethals, Seidel, and Shult (1978): If G is a graph with least eigenvalue not less than -2, then either G is a generalized line graph or G can be represented by one of the real root systems E_6, E_7, or E_8. Using this theorem, Doob and Cvetković (1979) have determined all connected graphs with least eigenvalue greater than -2. There are two infinite families and 571 other graphs. The first chapter is concerned with these theorems and other characterizations of graphs by their spectra, and in general spectral properties of various types of graphs. The second chapter is concerned with a family of highly regular graphs called distance-regular graphs. Eigenvalue techniques have been used to classify such graphs and to derive specific properties.

Chapter 3, the longest in the book, discusses several different topics from graph spectral theory in which there has been significant recent progress. These include graphical interpretations of the determinant and characteristic polynomial, spectra of graphs derived from other graphs using various transformations and operations, and the relationship between the automorphism group of a graph and eigenvalue multiplicities. In addition there is a

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section on bounding various graph invariants by using the spectrum of a graph. Included is a discussion of Lovász's recent (1979) determination of the Shannon capacity of the pentagon using spectral and other linear algebraic techniques.

In Chapter 4 the focus is on polynomials associated with a graph which are related to the characteristic polynomial. Most notable among these is the matching polynomial, which is the generating function for the number of matchings of different sizes in a graph. The matching polynomial of a graph is the same as the characteristic polynomial exactly when the graph is a forest, but Godsil (1981) has shown that the matching polynomial of a graph is a divisor of the characteristic polynomial of some forest. Other polynomials studied include the permanental polynomial and the characteristic polynomial of the Laplacian matrix of a graph.

Chapter 5 contains a review of over 200 papers published since 1978 that are concerned with applications of graph spectra to the natural sciences, primarily to chemistry. As is pointed out, the mathematical results in these papers are usually minor, but the applications to the discipline are often significant. The discussion in the book focuses on the mathematics and not the chemistry or physics.

The final chapter concerns the extension of graph spectral theory to infinite graphs. This of course gets involved with the spectral theory of operators in infinite dimensional Hilbert spaces. The book ends with an over 50 page table giving the spectrum of all graphs on seven vertices (tables giving the spectrum of graphs on less than seven vertices had been previously constructed).

For someone working in the area of graph spectral theory this book is a great resource. For someone who wants to see what has been going on in graph spectral theory in the last ten years and what some of the open problems are, the book is highly recommended.

REFERENCES

1. D. Cvetković, M. Doob, and H. Sachs, *Spectra of Graphs—Theory and Applications*, 2nd ed., Academic, New York, 1982.

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